Recitation 8

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Review

Determinants:

- $\det AB = \det A \det B$
- $\det A^T = \det A$
- square matrix A is invertible if and only if $\det A \neq 0$

Eigenvalues and eigenvectors: the following are equivalent

- λ is an eigenvector of a matrix A;
- $Av = \lambda v$ for some **non-zero** vector v;
- $(A - \lambda I)v = 0$ for some **non-zero** vector v;
- $(A - \lambda I)$ has non-zero null space;
- $(A - \lambda I)$ is not invertible;
- $\det(A - \lambda I) = 0$.

Problems

Problem 1. Is $\lambda = 4$ an eigenvalue of $A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$? Can you guess the other eigenvalue?

Problem 2. Compute determinant of the following matrix using cofactor expansion:

$$\begin{bmatrix} 0 & -1 & 2 \\ 0 & 2 & 1 \\ -1 & 3 & 3 \end{bmatrix}$$

Problem 3. Compute determinant of the following matrix using cofactor expansion:

$$\begin{bmatrix} 0 & 3 & 1 \\ -1 & 2 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

Now compute the same determinant using row reduction.

Problem 4. Compute determinant of the following matrix:

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 3 & -6 & -3 & -9 \\ -2 & -6 & 3 & 0 & 5 \\ 1 & 4 & 1 & 2 & -1 \\ 0 & 1 & -2 & -1 & -2 \end{bmatrix}$$

Do you want to try and compute this determinant using cofactor expansion?

Problem 5. Show that if a matrix A is invertible, then $\det A^{-1} = \frac{1}{\det A}$.

Problem 6. Suppose you know that $\det A = 2$ and $\det B = -1$ for some 3×3 matrices A, B. Compute the determinant of the matrix $2A^{-2}B^T A^{-1} B^2 A^T$.

Problem 7. Find eigenvalues of $A = \begin{bmatrix} 3 & 2 \\ 3 & 4 \end{bmatrix}$. Find the corresponding eigenvectors. Do they form a basis of $\mathbb{R}^2$?

Problem 8. Find eigenvalues of $A = \begin{bmatrix} 1 & 2 \\ -2 & 5 \end{bmatrix}$. Can you find a basis of $\mathbb{R}^2$ consisting of eigenvectors of A?

Problem 9. Find eigenvalues of the matrix

$$A = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

Problem 10. Find characteristic polynomial for the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Take a close look at the coefficients of that polynomial. Do you see anything familiar?

Problem 11. Find the coefficient of λ^2 and the constant coefficient of the characteristic polynomial for

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Hint: you don't really need to do any calculations, at least any long ones.